Learn fallacy of wage differentials with cohort data in 10 minutes

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This paper outlines "A fallacy of wage differentials: wage ratio in distribution [http://keijisaito.info/PDF/fallacy_wage_differential.pdf]" and "Do schools form human capital? distributional divide and cohort-based analysis in Japan [http://keijisaito.info/PDF/human_capital_cohort_japan.pdf]".[1] Even if you are unfamiliar with economics and mathematics, you can learn the key points in 10 minutes.

1. Economic Theories about Education

Across the world, education is perceived to be connected with getting a job and earning wages. Economics has two major theories about education.

Human capital theory : Education increases an individuals productivity and wages.

Signaling theory : Individuals can exhibit their productivity and diligence by entering and graduating from reputable educational institution.

Even if the data on wages and educational status are available, it is difficult to distinguish between the in-school effects from the pre-school productivities. Moreover, one theory does not violate the other. Previous attempts to determine which theory is more practical have been inconclusive. This paper provides the conclusion in the case of Japan.

2. Misleading Wage Differential: Wage Ratio between Educational Statuses

Apart from the economic theories about education, wage differentials have been a topic of debate in media and policymaking. A wage ratio is a basic index of the wage differential across educational statuses. The wage ratio is calculated by dividing the average wage of people with higher educational status by the average wages of those having lower educational status.[2] In most cases, the higher educational status means college graduates, while the lower one means high-school graduates. In other words, the primary wage ratio is $\frac{\text{average wage of college graduates}}{\text{average wage of high-school graduates}}$. The wage ratio is easy to interpret 1.2 as "20% higher" or "1.2 times". Rise in prices has no effect on the wage ratio. However, the wage ratios are misleading indexes of wage differentials.

[Example of the Wage Ratio]

Let me present the misleading point of the wage ratio with a simple example. Three men whose productivities are $A > B > C$, respectively come into the world. A is twice as productive as B, and A is three times as productive as C. Three men earn \$300, \$200, and \$100 respectively according to each productivity. An individual goes to higher education in order of their potential productivities. I tentatively assume that education has no effects on productivity and wage.

In the past, only A went to higher education. Nowadays, both A and B go to higher education. "How does the wage ratio $\left(= \frac{\text{average wage of higher educational status}}{\text{average wage of lower educational status}}\right)$ change?"

$$\textbf{past}: \text{ wage ratio} = \frac{A'\text{s wage}}{\frac{B'\text{s wage}+C'\text{s wage}}{2}} = \frac{\$300}{\frac{\$200+\$100}{2}} = \frac{\$300}{\$150} = 2$$

$$\textbf{now}: \text{ wage ratio} = \frac{\frac{A'\text{s wage}+B'\text{s wage}}{2}}{C'\text{s wage}} = \frac{\frac{\$300+\$200}{2}}{\$100} = \frac{\$250}{\$100} = 2.5$$

As a matter of course, wage of each individual remain constant because education has no effect. However, the wage ratio does increase from 2 to 2.5.

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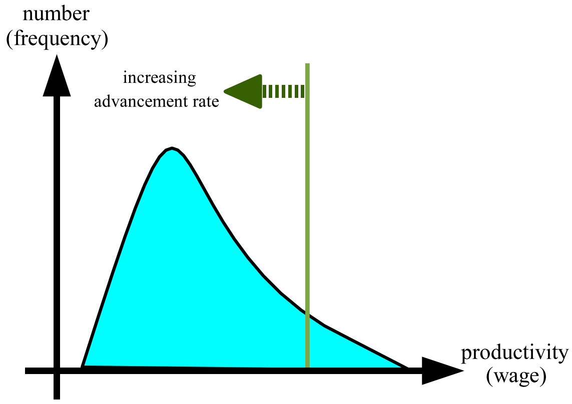
[1] Section 2 is from "A fallacy of wage differentials." Section 3 is from "Do schools form human capital?"

[2] Academic papers often take the logarithm. Monotonic transformation does not change the essence.

The example of $\{\$100, \$200, \$300\}$ is a simplified version of a uniform distribution that is high, middle, and low levels of productivity exist evenly. The increasing advancement rate necessarily increases the wage ratio in any uniform distribution. Meanwhile, it is natural to assume that the actual wage and productivity follow a mountain-shaped distribution as shown in the right figure.



In most mountain-shaped distributions, the wage ratio increases after an initial decline with an increase in the advancement rate. The below table shows a relationship between the advancement rate and the wage ratio that can be examined by hand calculation.

dividing line \ capability (number)	1 (2)	2 (3)	3 (3)	4 (1)	5 (1)	advancement rate	wage ratio	the logarithm of wage ratio
between 4 and 5	2.333				5	10%	2.143	0.762
between 3 and 4	2.125			4.5		20%	**2.118**	**0.750**
between 2 and 3	1.6		3.6			50%	2.250	0.811
between 1 and 2	1	3				80%	3.000	1.099

The minimum wage ratio cell is colored with yellow. In this numerical value, the minimum advancement rate (10%) brings the wage ratio to 2.143. The second-minimum advancement rate (20%) brings the minimum wage ratio to 2.118. The advancement rate of more than 50% increases the wage ratio. In summary, the wage ratio begins to increase reversely after a decline if the advancement rate increases. The wage ratio changes even if there are neither individual wage variations nor the effects of education.

For the sake of simplicity, the above examples have assumed that education has no effect on wage. Even if I assume that education has effects on wage, the main result is very similar to those in the case of no effect. If you would like to simulate ratio between averages with an arbitrary distribution and effect of education, use the Excel simulation: Simulation of Ratio between Averages[http://keijisaito.info/arc/Excel/average_ratio_distribution_en.xls]

We have hitherto assumed that people go to higher education strictly in the order of their potential productivity. However, the decision to pursue higher education depends not only on productivity or test scores but also on other factors like household income, location, character. Moreover, some people earn poor scores but earn a fortune and vice versa. The realistic assumption is that there is a positive correlation between the propensity to go for higher education and the potential wages of such people, although it is not a perfect one-to-one correspondence. However, if there is a positive correlation between them, the wage ratio of averages is the same as that in the case of one-to-one correspondence.[3]

The key points in the examples and simulations are as follows:

[1] Even if the individual wages are constant, the advancement rate changes the wage ratio.
[2] After an initial decline, the wage ratio increases in most mountain-shaped distributions.
[3] Even if education has actual effects on wages, the transition of the wage ratio is similar to that in the case of no effect.
[4] Even if a one-to-one correspondence does not exist between propensity to go for higher education and the potential wage, the wage ratio is the same to that in the case of one-to-one correspondence.

The advancement rate to higher education has been increasing in most countries. The changes in the wage ratio are natural outcomes in any case. Thus, the wage ratios are misleading indexes of wage differentials.

[3] Wage distributions across educational status overlap realistically in the correlation setting.

3. Data Analyses for Japan

This section analyzes the change in the wage ratio with data in Japan. The data has been collected through 28 consecutive annual surveys titled "Basic Survey on Wage Structures (BSWS)" from 1976 to 2003. The BSWS provides the most comprehensive information on wages in Japan.[4] The wage data for analyses are calculated by adding the annual special cash earnings to twelve times the contractual monthly cash earnings in June.[5] The BSWS categorized the levels of education into four classes:

(1) Graduates from junior high-schools
(2) Graduates from high-schools
(3) Graduates from higher professional schools and junior colleges
(4) Graduates from colleges or more

The wage ratio between (2) high-school graduates and (4) college graduates is considered as a primary indicator of the wage differential across educational statuses. At first, I tabulated the wage ratio of male employees ($=\frac{\text{average wage of male college graduates}}{\text{average wage of male high-school graduates}}$) as shown in five tables in page 4. Each table lines up from the upper-left line to the lower-right for every 5 surveys. Published BSWS records the average wages of five-year age brackets. I term the column birth bracket in accordance with age bracket of row. These five lined-up tables are based on five-year period birth brackets. Owing to this five-year arrangement, the birth brackets do not overlap in each table.

The educational advancement rate in Japan as well as in many countries has increased substantially in the 20th century. Schooling in youth virtually decides the share of educational status in each birth bracket. Fixing the birth bracket is almost equivalent to fixing the share of educational status regardless of the age bracket. Therefore, comparing the columns of the birth bracket enables the analysis of the effect of the educational advancement rate on the wage ratio.

I have marked the comparison on the immediate left column on page 4. There are two irregular movements in transition of birth bracket effects about 1953 and 1973.[6] Despite the irregular movements, on the whole, the birth bracket effects increase after a decline along with the generational progress. This transition seems to match [2] stated on page 2. However, one of the assumption on page 2 is not satisfied with the BSWS. The example on page 2 indicated the relationship between one dividing line and the wage ratio. On the other hand, the wage ratio obtained from the BSWS considers only two educational statuses, namely, college and high-school graduates, and it excludes the other two.

Even though I assume a mountain-shaped distribution, there is no general result with three dividing lines. A simple way to create one dividing line is by integrating the weighted average of employees. However, the weighted average method requires that the effects of education be negligible. As indicated by [3] on page 2, even if education has effects on wages, the transition of the wage ratio is similar to that in the case of no effect. It is very difficult to measure the actual effects of education by using only the wage ratios of averages without specifying the wage distribution.

However, there is a simple method to check the effects of education without specifying the wage distribution. Let me present the simple method with $\{A, B, C\}$ as on page 1. $\frac{A'\text{s wage}}{\frac{B'\text{s wage}+C'\text{s wage}}{2}}$ does not generally balance with $\frac{\frac{A'\text{s wage}+B'\text{s wage}}{2}}{C'\text{s wage}}$ regardless of the effect of education. Meanwhile, if education has no effect on wages, there are some ratios that always balance regardless of the position of the dividing line. The ratios are simply $\frac{A'\text{s wage}}{B'\text{s wage}}$ and $\frac{B'\text{s wage}}{C'\text{s wage}}$. If education has no effect, these ratios always remain constant. Conversely, let me assume that education has a positive effect and B goes to higher education. In this

[4] The Ministry of Health, Labour and Welfare of Japan compiles the BSWS. Each survey is based on approximately 1.5 million employees. The BSWS is one of the most reliable and long-running wage data in the world.

[5] It represents annual wage. Even though I use hourly wages instead of the annual wages, the principal results in this paper are robust.

[6] These irregular movements are mainly from the oil shocks and bubble economy bursts. These indicate that business climates existing at the time of the graduation of students have persistent effects on the wages in Japan.

The Wage Ratio between College and High-School Graduates: Male

The Wage Ratio between College and High-School Graduates: Male = average wage of male college graduates / average wage of male high-school graduates

Compared with each immediate left birth bracket, two or more all comparable ratios

- increase : ↗
- decrease : ↘
- are almost the same (the differences are less than 2%) : →
- fluctuate with the age : ?

final numbers: 1 and 6 — from the upper left corner 1976, 1981, 1986, 1991, 1996, 2001 survey

age bracket \ birth bracket (cohort)	1911–1916	1916–1921	1921–1926	1926–1931	1931–1936	1936–1941	1941–1946	1946–1951	1951–1956	1956–1961	1961–1966	1966–1971	1971–1976	1976–1981
20–24 years old									92.0%	92.2%	96.7%	98.9%	96.3%	99.8%
25–29 years old								106.4%	102.5%	110.3%	111.0%	112.5%	111.1%	
30–34 years old							115.2%	116.5%	115.6%	120.6%	122.9%	125.8%		
35–39 years old						129.9%	121.7%	126.7%	122.7%	130.4%	133.4%			
40–44 years old					142.2%	138.8%	131.6%	132.9%	129.8%	137.9%				
45–49 years old				159.0%	151.4%	149.3%	135.9%	139.9%	137.4%					
50–54 years old			167.6%	161.4%	158.4%	155.7%	145.2%	145.8%						
55–59 years old		163.1%	170.7%	170.4%	161.0%	156.6%	148.8%							

final numbers: 2 and 7 — from the upper left corner 1977, 1982, 1987, 1992, 1997, 2002survey

age bracket \ birth bracket (cohort)	1912–1917	1917–1922	1922–1927	1927–1932	1932–1937	1937–1942	1942–1947	1947–1952	1952–1957	1957–1962	1962–1967	1967–1972	1972–1977	1977–1982
20–24 years old									90.9%	94.0%	98.1%	99.1%	95.6%	100.5%
25–29 years old								107.1%	103.7%	111.1%	111.8%	112.3%	114.0%	
30–34 years old							113.8%	116.5%	116.2%	120.7%	123.9%	126.7%		
35–39 years old						128.4%	121.2%	127.1%	124.1%	129.9%	134.8%			
40–44 years old					139.7%	136.5%	131.3%	135.5%	130.2%	140.9%				
45–49 years old				158.6%	151.1%	146.7%	137.7%	139.3%	138.0%					
50–54 years old			166.5%	166.3%	158.1%	152.5%	140.9%	147.6%						
55–59 years old		170.7%	175.1%	172.6%	156.8%	150.8%	152.2%							

final numbers: 3 and 8 — from the upper left corner 1978, 1983, 1988, 1993, 1998, 2003survey

age bracket \ birth bracket (cohort)	1913–1918	1918–1923	1923–1928	1928–1933	1933–1938	1938–1943	1943–1948	1948–1953	1953–1958	1958–1963	1963–1968	1968–1973	1973–1978	1978–1983
20–24 years old									90.7%	96.2%	97.7%	99.6%	97.9%	102.5%
25–29 years old								105.1%	104.3%	111.2%	112.1%	112.6%	115.4%	
30–34 years old							115.0%	116.2%	116.5%	121.5%	125.1%	124.7%		
35–39 years old						127.0%	122.6%	125.3%	124.9%	132.5%	136.8%			
40–44 years old					141.0%	134.6%	130.2%	134.6%	135.1%	139.3%				
45–49 years old				152.9%	149.2%	142.5%	139.5%	140.3%	139.1%					
50–54 years old			168.6%	162.7%	156.1%	150.1%	146.6%	142.9%						
55–59 years old		174.2%	172.2%	166.4%	163.3%	156.3%	147.9%							

final numbers: 4 and 9 — from the upper left corner 1979, 1984, 1989, 1994, 1999survey

age bracket \ birth bracket (cohort)	1914–1919	1919–1924	1924–1929	1929–1934	1934–1939	1939–1944	1944–1949	1949–1954	1954–1959	1959–1964	1964–1969	1969–1974	1974–1979	
20–24 years old									92.0%	96.0%	99.9%	98.5%	98.6%	
25–29 years old								103.8%	105.2%	111.8%	111.5%	112.1%		
30–34 years old							115.1%	114.5%	118.4%	121.1%	125.2%			
35–39 years old						124.3%	124.5%	125.6%	127.1%	132.0%				
40–44 years old					139.0%	131.9%	132.9%	133.5%	134.0%					
45–49 years old				151.0%	147.7%	141.7%	140.5%	139.6%						
50–54 years old			167.3%	158.4%	157.9%	148.5%	146.4%							
55–59 years old		169.9%	174.9%	170.8%	162.5%	154.1%								

final numbers: 5 and 0 — from the upper left corner 1980, 1985, 1990, 1995, 2000survey

age bracket \ birth bracket (cohort)	1915–1920	1920–1925	1925–1930	1930–1935	1935–1940	1940–1945	1945–1950	1950–1955	1955–1960	1960–1965	1965–1970	1970–1975	1975–1980	
20–24 years old									92.1%	97.1%	99.0%	97.0%	98.9%	
25–29 years old								102.4%	107.5%	111.4%	112.3%	109.4%		
30–34 years old							114.8%	114.1%	119.0%	121.9%	125.2%			
35–39 years old						122.7%	125.5%	123.7%	128.2%	132.2%				
40–44 years old					139.0%	130.0%	133.6%	132.2%	136.0%					
45–49 years old				151.5%	148.3%	140.8%	140.8%	135.7%						
50–54 years old			164.9%	156.5%	156.1%	145.9%	146.3%							
55–59 years old		166.7%	174.9%	165.6%	157.8%	148.7%								

＊ The birth brackets are from July 1 of the first year to June 30 of the last year.

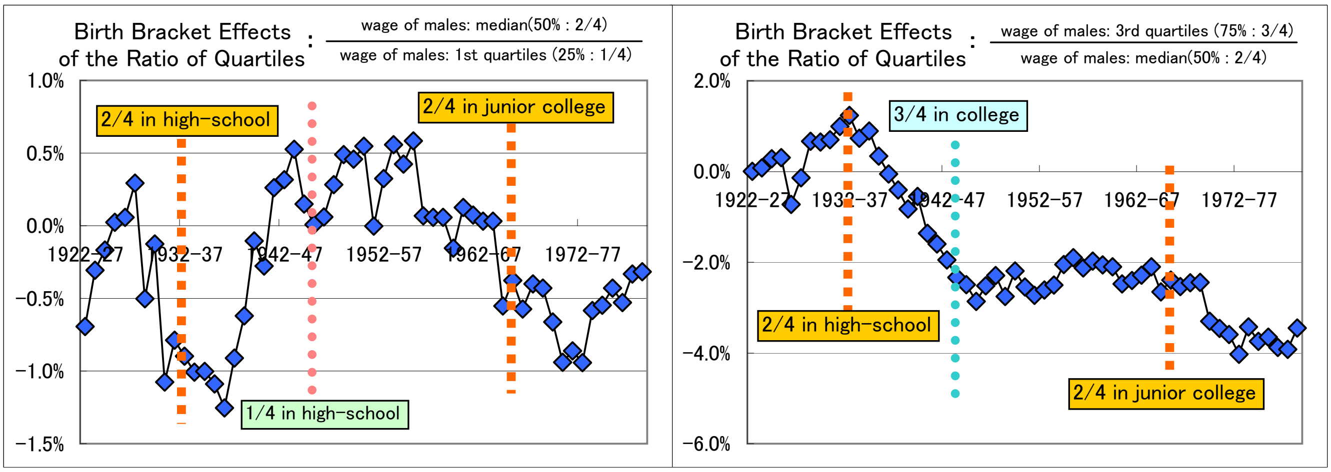
assumption, the simple result is that the raised value of the denominator decreases $\frac{A'\text{s wage}}{B'\text{s wage}}$, while the raised value of the numerator increases $\frac{B'\text{s wage}}{C'\text{s wage}}$. The quartiles of wages are useful to check to the effects of education on this idea. The BSWS records quartiles of wage without the separation of educational statuses: the first quartile (25%), the median (50%), and the third quartile (75%).

The ratios of quartiles are calculated by dividing the wage of higher quartile by that of the lower quartile. I decompose the ratio of quartiles into effects of the age and birth brackets. The left lower graph indicates the birth bracket effects of male employees between the first quartile (25%) and the median (50%). The right lower graph indicates the birth bracket effects of male employees between the median (50%) and the third quartile (75%). The upper side of the graphs represents the widening of the gap. I draw dotted lines at the points where the share of educational statuses intersect 25%, 50%, and 75% in the graphs.[7] For example, 2/4 in high-school means median (50%) male employee is a junior high school graduate at the left (before) side of the dotted line and is a high-school graduate at the right (after) side of the dotted line.

On the whole, the ratios of quartiles decrease when the educational status of the higher quartile changes and increase when the educational status of the lower quartile changes.[8] Most of the people who do not go to higher education get jobs earlier than the ones going to it. Productivities and wages are raised if skills are acquired at work. If the effects of education and that of job are the same, the ratios of quartiles are independent of the share of educational status. However, the lower graphs imply that the effects of education are generally less than those of job. In other words, higher education has net negative effects on productivities which are reflected in wages.

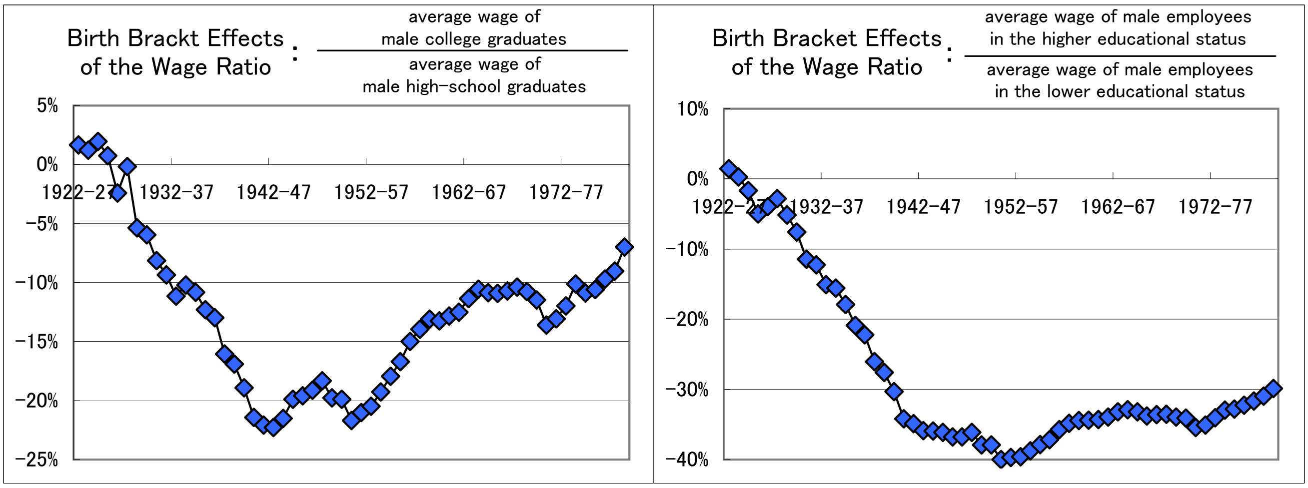
Though these graphs imply net negative effects of education, there may be variations that are irrelevant to the effects of education, such as the trend of the decreasing wage gap after the birth bracket of the 1960's. In addition, vertical axes indicate that the total variations are of a few percentage points. If the net effects of education are close to zero, the weighted average method is justified as an acceptable approximation.

The average wage of the employees belonging to lower educational status is calculated from the weighted average of (1) graduates from junior high-schools and (2) graduates from high-schools. In the same manner, the average wage of the employees belonging to higher educational status is calculated from the weighted average of (3) graduates from higher professional schools and junior colleges and (4) graduates from colleges



[7] If the (upper) numerator quartile gets into the next educational status, a label is attached at the head of the dotted line. If the (lower) denominator quartile gets into the next educational status, a label is attached at the bottom of the dotted line.

[8] The ratios of quartiles change just before the dotted lines pass over the quartiles. This would be consistent if the effects of education are net negative.



or more. The upper left graph shows the transition of the birth bracket effects between college and high-school graduates based on the data on page 4. The upper right graph shows the transition of the birth bracket effects between the higher educational status and the lower one, achieved by using the weighted average.

The transition of the upper right graph is smoother than the one on the upper left. The spreading time of graduation alleviates the initial effect of new graduates. In addition, taking account of the entire distribution stabilizes the transition based on mountain-shaped distribution.[9]

Only the birth bracket effects of columns and the age bracket effects of rows can account for more than 99% of the wage ratio variations. It is unlikely that there are substantial variations in the wage structures at some point. The main reason for changes in wage differentials in Japan is a statistical trick that results from the distributional divide.

Here are the key aspects of the data analyses for Japan.

[1] The primary wage ratio between college and high-school graduates increases after a decline.
[2] The business climate at the time of the graduation has persistent effects on wages.
[3] The net effects of education on wages are negative or even close to zero.
[4] The wage ratio transitions follow the entire distribution rule including graduates from junior high-schools and graduates from higher professional schools and junior colleges.
[5] The main reason for the variations in wage differentials is a statistical trick that results from the distributional divide.

4. Concluding Remarks

Section 1 introduces the question of whether the human capital theory or the signaling theory is more practical. Since the results imply net human capital losses, it is safe to state that the signaling theory is more practical at least in the case of Japan. In general, higher education is not suitable for nurturing of employees.

Even if the signaling theory is practical, going to higher education would be profitable from the viewpoint of households. In order to demonstrate his/her productivity, an individual must go to higher education even if productivity decreases. Though going to higher education is profitable from the view point of households, society suffers from losses in job skill acquisition and economic growth. Indeed, education and learning are virtues in most countries. Schooling possess diverse aspects of culture. However, we have to rethink and discuss about education and schooling.

[9] Net negative effects of education and the smooth transition of the weighted wage ratio are similar between male employees and female ones.